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COMMENT

Self-attracting walk on lattices

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Abstract. We have studied a model of self-attracting walk proposed by Sapozhnikov using the Monte Carlo method. The mean-square displacement $\langle R^2(t)\rangle \sim t^{2\nu}$ and the mean number of visited sites $\langle S(t)\rangle \sim t^k$ are calculated for one-, two- and three-dimensional lattices. In one dimension, the walk shows diffusive behaviour with $\nu = k = \frac{1}{2}$. However, in two and three dimensions, we observed a non-universal behaviour, i.e. the exponent ν varies continuously with the strength of the attracting interaction.

There is great interest in random walks (RWs) and interacting walks [1–3]. The mean-square displacement of a RW, $\langle R^2(t)\rangle$ follows a power-law $\langle R^2(t)\rangle \sim t^{2\nu}$. The ordinary RW is diffusive, with $\nu = \frac{1}{2}$, in all dimensions. For a walk with repulsion, such as self-avoiding walk (SAW), the exponent ν is greater than $\frac{1}{2}$. A RW on a fractal is anomalous with $\nu < \frac{1}{2}$ [2, 3]. Various models of interacting walks have been studied, such as true SAWs [4, 5], generalized true SAWs [6], the Domb–Joyce model [7], and an interacting walk with a weight factor p for each new site that the random walker visits [8, 9]. A comparative study of interacting RW models was performed by Duxbury *et al* [10, 11].

Recently, Sapozhikov proposed a generalized walk in which the probability for the walker to jump to a given site is proportional to $p = \exp(-nu)$, where $n = 1$ for the sites visited by the walker at least once and $n = 0$ for other sites [12]. If $u < 0$, the walker is attracted to its own trajectory. This walk is called a self-attracting walk (SATW). Monte Carlo studies have suggested that $\nu < \frac{1}{2}$ for $u = -1$ and $u = -2$ in two dimensions and $\frac{1}{4} < \nu < \frac{1}{3}$ on three dimension [12]. However, Aarão Reis obtained non-universal behaviour of the SATW in 1–4 dimensions using an exact enumeration method [13]. Prasad *et al* concluded that the SATW in one dimension is diffusive [14].

In this comment we report results of a Monte Carlo simulation for the SATW in one, two and three dimensions. We find that SATW in one dimension is diffusive. However, SATW in two and three dimensions shows non-universal behaviour. Monte Carlo simulations were performed on a one-dimensional lattice (10^6 steps with 2000 configurational averages for each parameter u), a square lattice (10^6 steps with 2000 averages), and a cubic lattice (10^5 steps with 2000 averages). The lattice sizes used in this simulation were $L = 10^6$ (1D), 1024×1024 (2D) and $200 \times 200 \times 200$ (3D). We always used periodic boundary conditions. We calculated the mean-square displacement $\langle R^2(t)\rangle$ and the mean number of sites visited $\langle S(t)\rangle$.

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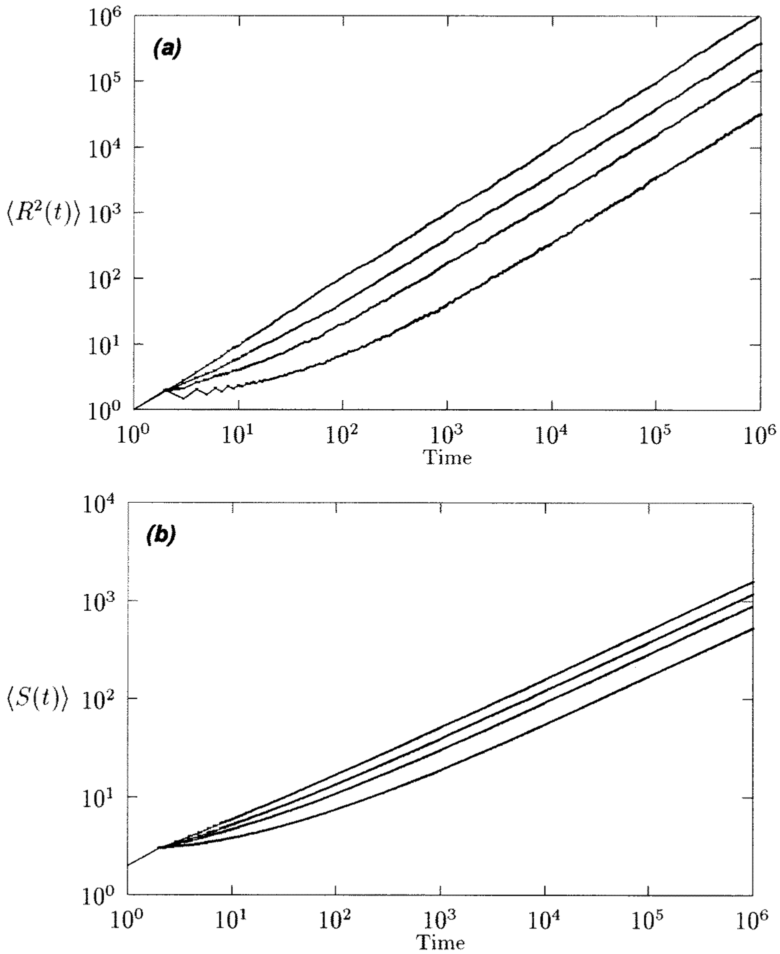

Figure 1. (*a*) Log–log plot of the mean-square displacement versus time; (*b*) log–log plot of the mean number of visited sites versus time for a one-dimensional SATW for $u = 0$ (top curve), -0.5, -1.0, -2.0 (bottom curve).

Figure 1(*a*) shows the log–log plot of the mean-square displacement against time. We obtained the exponent by a least-square fit: $\nu = 0.500(7)$ for $u = 0$, $\nu = 0.500(9)$ for $u = -0.5$, $\nu = 0.499(8)$ for $u = -1.0$, and $\nu = 0.498(9)$ for $u = -2.0$. All the lines are parallel in the large-time limit. These results mean that a SATW in one dimension is diffusive and support the conclusions of Prasad *et al* [14]. This diffusive behaviour is further supported by the results of the mean number of visited sites, $\langle S(t)\rangle$, in figure 1(*b*). The mean number of visited sites $\langle S(t)\rangle$ shows the power-law behaviour as $\langle S(t)\rangle \sim t^k$ with $k = 0.499(7)$ for $u = 0.0$, $u = 0.499(4)$ for $u = -0.5$, $k = 0.499(8)$ for $u = -1.0$, and $k = 0.498(5)$ for $u = -2.0$. At short times the slopes of the mean-square displacement depend on the parameter u. However, for large times all the lines are parallel and give the same slope regardless of the parameter u. Aarão Reis concluded that the exponent ν in one dimension decreases continuously when u decreases, by the exact enumeration calculation up to $t = 30$ [13]. But their conclusion is not correct because the time steps are too short to reach the asymptotic behaviour, and the log–log plot of $\langle R^2(t)\rangle$ has curvature at short times.

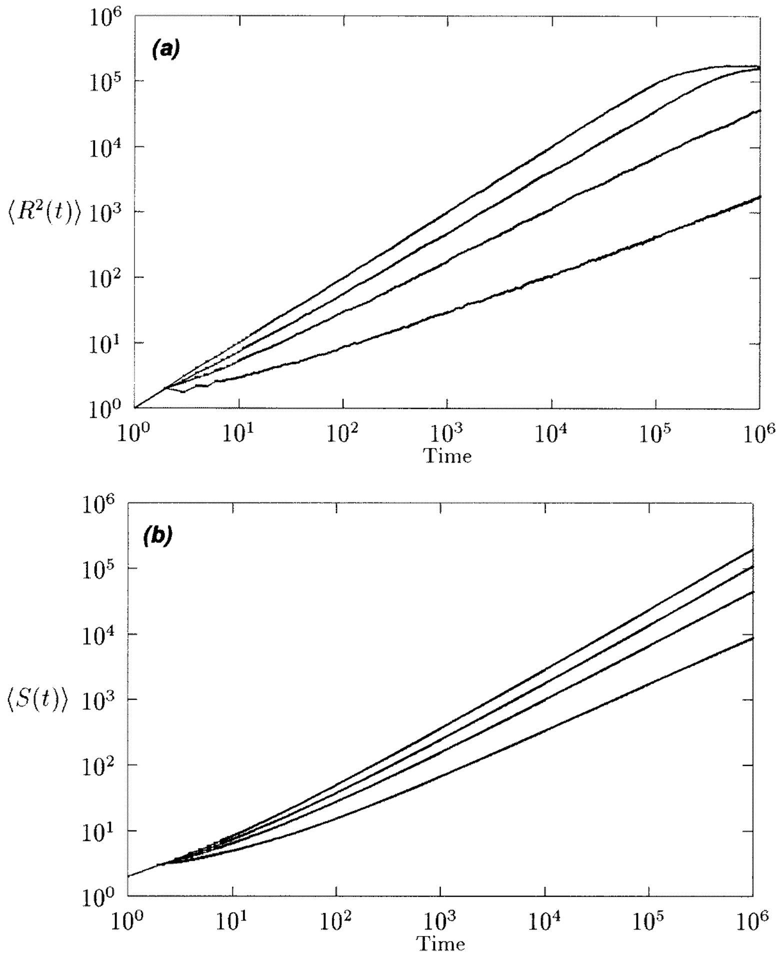

Figure 2. (a) Log–log plot of the mean-square displacement versus time; (b) log–log plot of the mean number of visited sites versus time for a two-dimensional SATW for $u = 0$ (top curve), -0.5, -1.0, -2.0 (bottom curve).

The log–log plot of the mean-square displacement and the mean number of visited sites of a two-dimensional SATW versus time are shown in figures 2(a) and (b), respectively. The appearance of the plateau region for large times is due to the finite size of the substrate. In this region the walk touches the boundary. The exponents ν were obtained as $\nu = 0.500(3)$ for $u = 0$, $\nu = 0.472(4)$ for $u = -0.5$, $\nu = 0.404(5)$ for $u = -1.0$, and $\nu = 0.300(9)$ for $u = -2.0$. Our results are consistent with those of Aarão Reis [13], Sapozhnikov [12] and Lee [15]. Values of the exponents ν decrease continuously when u decreases. We cannot observe the critical value u_c proposed by Sapozhnikov so that $\nu = \frac{1}{2}$ for $0 < u < u_c$ [12]. The mean number of visited sites $\langle S(t)\rangle$ shows the logarithmic correction for random walk ($u = 0$ case) [1, 2]. The slopes of the log–log plots of $\langle S(t)\rangle$ decrease continuously when u decreases.

Figures 3(a) and (b) show a log–log plot for the mean-square displacement and the mean number of visited sites for a three-dimensional SATW in a cubic lattice. The plateau region is also due to the finite size of the substrate. The exponents ν were obtained as

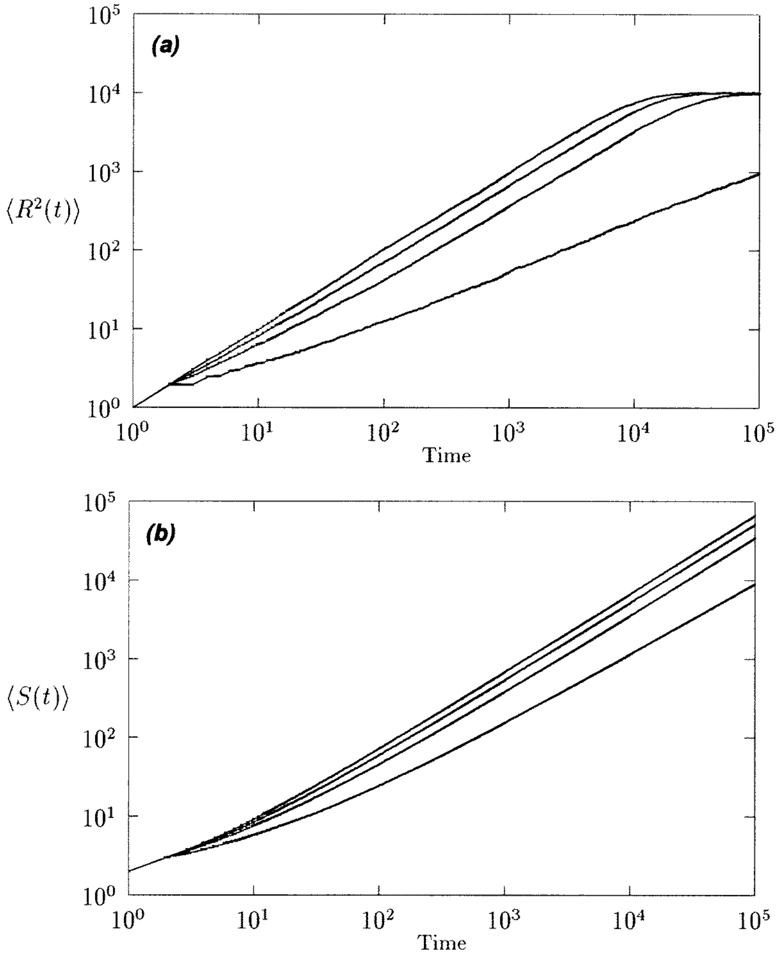

Figure 3. (*a*) Log–log plot of the mean-square displacement versus time; (*b*) log–log plot of the mean number of visited sites versus time for a three-dimensional SATW for $u = 0$ (topmost curve), -0.5, -1.0, -2.0 (bottom curve).

$\nu = 0.500(2)$ for $u = 0$, $\nu = 0.485(4)$ for $u = -0.5$, $\nu = 0.466(5)$ for $u = -1.0$, and $\nu = 0.294(6)$ for $u = -2.0$. The exponents ν also decrease continuously when u decreases. These results are also consistent with those of Aarão Reis [13], but they are not consistent with the prediction of Sapozhnikov that $\frac{1}{4} < \nu < \frac{1}{3}$. In Sapozhnikov's argument there is an ambiguity regarding the scaling between the bulk cluster and the boundary cluster visited by the walk. We obtained the exponents k as $k = 0.996(5)$ for $u = 0$, $k = 0.993(2)$ for $u = -0.5$, $k = 0.991(5)$ for $u = -1.0$, and $k = 0.904(6)$ for $u = -2.0$. For a RW, $k = 1$ in three dimensions [1, 2]. We observed that the values of k decrease slowly when u decreases. These Monte Carlo results are also in good agreement with the observations of Aarão Reis [13]. The continuous decrease of the exponent ν and k in three dimensions is further confirmed by the simulation at the u-values between $u = -1.0$ and $u = -3.0$. We obtained the exponents as $\nu = 0.436(4)$, $k = 0.970(3)$ for $u = -1.5$, $\nu = 0.240(5)$, $k = 0.806(3)$ for $u = -2.5$, and $\nu = 0.190(8)$, $k = 0.776(5)$ for $u = -3.0$. These results support the idea that there are no crossover behaviours in the range $-3.0 \leqslant u \leqslant 0$.

We calculated the exponents ν and k in one, two and three dimensions by Monte Carlo simulations. We have concluded that a SATW in one dimension is diffusive. This observation supports the recent calculation of Prasad *et al* [14]. However, in two and three dimensions the exponents ν decrease continuously when u decreases. These non-universal behaviours are consistent with those of Aarão Reis [13]. Our observations are limited on regular lattices. One still has to explore the scaling behaviour on general lattices such as fractals.

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